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## THE DYNAMIC EXPANSION OF A SPHERICAL CAVITY IN AN ELASTIC-PERFECTLY-PLASTIC MATERIAL

by

Chi-Hung Mok

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February 1967

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REPORT NO. 1357

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THE DYNAMIC EXPANSION OF A SPHERICAL CAVITY IN AN  
ELASTIC-PERFECTLY-PLASTIC MATERIAL

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Exterior Ballistics Laboratory

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REPORT NO. 1357

CHMok/cr  
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ABSTRACT

It has been shown that a finite-difference numerical technique can be used to solve mixed initial-and boundary-value problems involving high-speed elastic-plastic flow with spherical symmetry. Numerical solutions for the dynamic expansion of a spherical cavity under a constant pressure are presented to demonstrate the nature and capability of the numerical scheme. The solution for an elastic material agrees closely with the exact solution. The solution for an elastic-perfectly-plastic material has confirmed Green's prediction concerning the motion of the elastic-plastic boundary. At large times, the asymptotic solution of the dynamic problem is different from the quasi-static solution. This result indicates that the quasi-static approximation may not hold in dynamic plasticity. A non-linear dependence of the plastic solution on the boundary condition is also observed in the results.

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## 1. INTRODUCTION

The problem of high-speed elastic-plastic flow with spherical symmetry is of interest for two reasons. First, it has practical application in underground explosions and in the detonation of high-explosives in solids; secondly, with little increase in the mathematical difficulties from the uniaxial flow, the simple geometry of this problem allows the study of an elastic-plastic flow involving the bi-axial state of stresses and inertia forces. Perhaps this second characteristic is equally interesting to both an applied mathematician and an experimentalist who is concerned with obtaining information of the dynamic yield properties of a material under bi-axial state of stresses. In the past, many authors have investigated this problem; the reader may refer to a paper by Hopkins<sup>1\*</sup> for a general review on the current status of research in this subject. Most of the published work is concerned with small deformations. The materials investigated include the idealized elastic-perfectly-plastic and elastic-linear-work-hardening materials. However, up to present, because of the mathematical difficulties involved in treating two-phase flows, there existed no analytical solution which describes satisfactorily all the phases of the impact process. As pointed out in Section 2 below, only at the very beginning of an impact is the location of the elastic-plastic boundary known and a simple analytic description of the flow available. However, for the major duration of the impact process, the description of the flow seems to rely upon a numerical solution. The purpose of this report is to show that this problem can be solved for various initial and boundary conditions by using a finite difference technique. The elastic-plastic solution obtained for the expansion of a spherical cavity under constant internal pressure has demonstrated some interesting facts concerning the impact phenomena. It has confirmed Green's<sup>1,2</sup> results regarding the motion of the elastic-plastic boundary and also has exhibited the inadequacy of the quasi-static approximation

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\* Superscript numbers denote references which may be found on page 30.

in dynamic plasticity. Numerical solutions of spherical elastic-plastic flow have also been obtained by Davids et al.<sup>3</sup> and by Friedman et al.<sup>4</sup>

## 2. BASIC EQUATIONS

Referring to a spherical coordinate system,  $r$ ,  $\theta$  and  $\phi$ , with the origin at the center of the cavity, only the equation of motion in the  $r$  direction will not vanish identically because of the spherical symmetry considered here; hence,

$$\frac{\partial \sigma_r}{\partial r} + 2 \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial v}{\partial t} . \quad (1)$$

In the equation,  $r$  is the Lagrangian position of a particle;  $\rho$  is the mass density of the material in the underformed state;  $v$  the radial velocity of the particle, and  $\sigma_r$  and  $\sigma_\theta$  are respectively the normal stress components in the radial and the circumferential directions. The stresses are engineering stresses calculated with reference to the undeformed state. This report considers only small deformations; hence, differences between the engineering and true values of stress and strain will be negligible.

The material is considered to be elastic, perfectly plastic with a constant yield stress  $\sigma_0$  in both simple compression and tension. Both von Mises and Tresca yield conditions reduce to the form<sup>5</sup>

$$\sigma_r - \sigma_\theta = \pm \sigma_0 . \quad (2)$$

The material will deform plastically when this equation is satisfied and when

$$(\sigma_r - \sigma_\theta) (\dot{\sigma}_r - \dot{\sigma}_\theta) = 0 . \quad (3)$$

Otherwise, when

$$|\sigma_r - \sigma_\theta| < \sigma_0 , \quad (4)$$

or when

$$\sigma_r - \sigma_\theta = \pm \sigma_0$$

but

(5)

$$(\sigma_r - \sigma_\theta) (\dot{\sigma}_r - \dot{\sigma}_\theta) < 0 ,$$

the material behaves as an elastic body.

For the elastic regime, the material obeys Hooke's law. Expressing the radial and circumferential strains,  $\epsilon_r$  and  $\epsilon_\theta$  respectively, in terms of the radial displacement  $u$ ,

$$\epsilon_r = \frac{\partial u}{\partial r}$$

$$\epsilon_\theta = \frac{u}{r} , \quad (6)$$

Hooke's law in a differential form can be written as

$$\frac{\partial \sigma_r}{\partial t} - 2\nu \frac{\partial \sigma_\theta}{\partial t} - E \frac{\partial v}{\partial r} = 0 \quad (7)$$

$$\nu \frac{\partial \sigma_r}{\partial t} - (1 - \nu) \frac{\partial \sigma_\theta}{\partial t} + \frac{E}{r} v = 0 .$$

These two equations and the equation of motion, Equation (1), must be solved simultaneously to obtain the elastic solution.

For most materials, under moderate pressures, no appreciable plastic volume change will occur. Hence, the elastic relation between the mean hydrostatic stress,  $(\sigma_r + 2\sigma_\theta)/3$  and the dilatation  $\partial u/\partial r + 2u/r$

can be used for describing the plastic flow. Using Hooke's law and relating  $\sigma_r$  to  $\sigma_\theta$  by the yield condition, the relation expressed in a differential form is

$$\frac{1 - 2\nu}{E} \frac{\partial \sigma_r}{\partial t} - 1/3 \frac{\partial v}{\partial r} - 2/3 \frac{v}{r} = 0 . \quad (8)$$



This equation must be solved simultaneously with the yield condition, Equation (2), and the equation of motion, Equation (1), to provide results for the plastic flow.

A convenient representation of the solutions of these equations can be obtained by introducing the following dimensionless variables,

$$\begin{aligned}
 x &= \frac{r}{r_o} \\
 R &= \frac{\sigma}{\sigma_o} r \\
 \Theta &= \frac{\sigma}{\sigma_o} \theta \\
 \bar{\sigma} &= \frac{\sigma_o}{E} \\
 \tau &= \frac{c_d t}{r_o}
 \end{aligned} \tag{9}$$

and

$$V = \frac{v}{c_d \bar{\sigma}}$$

where  $r_o$  is the radius of the cavity and  $c_d = [(1 - \nu)(E/\rho)/(1 - \nu - 2\nu^2)]^{1/2}$  is the dilatational wave speed in an elastic solid. In terms of these dimensionless variables, the simultaneous partial differential equations for the dynamic problem are

$$\begin{aligned}
 \frac{\partial R}{\partial x} - \frac{1-\nu}{1-\nu-2\nu^2} \frac{\partial V}{\partial \tau} + \frac{2}{x} (R - \Theta) &= 0 \\
 \frac{\partial R}{\partial \tau} - 2\nu \frac{\partial \Theta}{\partial \tau} - \frac{\partial V}{\partial x} &= 0 \\
 \nu \frac{\partial R}{\partial \tau} - (1 - \nu) \frac{\partial \Theta}{\partial \tau} + \frac{1}{x} V &= 0
 \end{aligned} \tag{10}$$

and

$$\frac{\partial R}{\partial x} - \frac{1-\nu}{1-\nu-2\nu^2} \frac{\partial V}{\partial \tau} \pm \frac{2}{x} = 0$$

$$(1 - 2\nu) \frac{\partial R}{\partial \tau} - \frac{1}{3} \frac{\partial V}{\partial x} - \frac{2}{3} \frac{V}{x} = 0 \quad (11)$$

$$\Theta = R \pm 1 .$$

Equation (10) will apply for elastic deformation; i.e., when

$$|R - \Theta| < 1 \quad (12)$$

or when

$$R - \Theta = \pm 1 \quad (13)$$

but

$$(R - \Theta) (\dot{R} - \dot{\Theta}) < 1 .$$

Equation (11) will hold for the plastic flow, when

$$R - \Theta = \pm 1$$

and

$$(R - \Theta) (\dot{R} - \dot{\Theta}) = 0 . \quad (14)$$

The  $\pm$  signs are for the situations when  $R - \Theta = \pm 1$  respectively.

In Equations (10) and (11), only one parameter, the Poisson's ratio  $\nu$  appears in the coefficients. The equations written in this form provide a clearer picture of the dependence of the solution on various parameters of material properties. For a stress boundary condition, the stress distribution around a plastically deformed cavity at any instant depends only on the Poisson's ratio and the ratio between the applied pressure to the yield stress of the material.

Equations (10) and (11) are two sets of first order linear hyperbolic partial differential equations. Their solutions can be obtained by combining two waves associating with a displacement function as described

by Hunter.<sup>6</sup> However, this method requires *a priori* a knowledge of the position of the boundary of the elastic and plastic regions of the material which can be obtained only in few instances. Green,<sup>1,2</sup> has proved that when there is a discontinuity in the magnitude of the radial strain  $\epsilon_r$  across the elastic-plastic boundary, the boundary will travel with a constant velocity  $c_p = [E/3 \rho (1 - 2\nu)]^{1/2}$ , and the position of the boundary is, therefore, known. However, with the rapid attenuation of a diverging spherical wave, this discontinuity in radial strain can be maintained only near the very beginning of an impact. Therefore, other techniques must be used to obtain the solution which holds for the major portion of the impact.

### 3. NUMERICAL SCHEMES

In this report, a finite difference scheme proposed by Lax<sup>7</sup> will be used to obtain a solution of the equations derived above. The original scheme has been shown suitable for solving initial value problems of hyperbolic partial differential equations in hydrodynamics. The scheme is extended here for mixed boundary-and initial-value problems with regions of solutions governed by different sets of equations. The results obtained using this scheme are compared with known analytical solutions for the case of elastic waves. The accuracy of the numerical results appears to be satisfactory. An appendix to this report describes the computer program used to obtain the present results.

The finite difference scheme is for any set of partial differential equations assuming the conservation form; i.e.,

$$\frac{\partial F}{\partial x} + \frac{\partial G}{\partial t} + H = 0 , \quad (15)$$

where  $F$ ,  $G$  and  $H$  are functions of the dependent and independent variables. The numerical scheme replaces  $\partial F/\partial x$  by a central difference quotient; i.e.,

$$\frac{F_{n+1,k} - F_{n-1,k}}{2\Delta x} \quad (16)$$

where  $\Delta x$  denotes an increment in  $x$ ;  $F_{n+1,k}$  designates the value of  $F$  at the grid point located at distances  $(n+1) \Delta x$  and  $k\Delta\tau$  from the origin in the  $x$  and  $\tau$  plane. The time derivative  $\partial G/\partial\tau$  is substituted by a forward difference quotient,

$$\frac{G_{n,k+1} - \frac{1}{2}(G_{n+1,k} + G_{n-1,k})}{\Delta\tau} \quad (17)$$

Finally, the function  $H$  is replaced by the average value at two neighboring grid points at the time  $k\Delta\tau$ ,

$$\frac{1}{2}(H_{n+1,k} + H_{n-1,k}) \quad (18)$$

The scheme is equivalent to adding into the original equation a term which is due to an artificial viscosity of the magnitude of  $(\Delta x)^2/2\Delta\tau$ . The stability and convergence of this scheme has been discussed by Lax.<sup>7</sup> The stability criterion is similar to the Courant-Friedrichs-Lewy condition, i.e.,  $\Delta\tau/\Delta x < c$  where  $c$  is a quantity which may depend on the variables in the problem.

The present equations, Equations (10) and (11), are in the conservative form and, hence, can be solved numerically with this method. As an example, the finite difference equation for the first equation of (10) is obtained; i.e.,

$$\begin{aligned} \frac{R_{n+1,k} - R_{n-1,k}}{2\Delta x} - \frac{1-v}{1-v-2v^2} \frac{V_{n,k+1} - \frac{1}{2}(V_{n+1,k} + V_{n-1,k})}{\Delta\tau} \\ + \frac{R_{n+1,k} - \Theta_{n+1,k}}{x_{n+1,k}} + \frac{R_{n-1,k} - \Theta_{n-1,k}}{x_{n-1,k}} = 0 . \end{aligned}$$

After being converted to finite difference equations, the three equations in (10) and in (11) will provide three algebraic equations sufficient for solving the values of the dependent variables at  $\tau = (k+1)\Delta\tau$ ;

namely,  $R_{n,k+1}$ ,  $V_{n,k+1}$  and  $\Theta_{n,k+1}$  from the known data at a previous time  $k\Delta\tau$ ; namely,  $R_{n-1,k}$ ,  $\Theta_{n+1,k}$ ,  $\Theta_{n-1,k}$ ,  $V_{n+1,k}$  and  $V_{n-1,k}$ . At each point, the finite difference equations for elastic deformations, Equation (10), will be first used to obtain  $R_{n,k+1}$  and  $\Theta_{n,k+1}$ . If the difference between them  $|R_{n,k+1} - \Theta_{n,k+1}|$  is greater than unity, then the calculations will be revised by using the finite difference equations for plastic flow which are derived from Equations (11).

The above scheme can be applied very conveniently at points other than those on the boundary. However, when the central point  $x_n$  is on the cavity surface, the point  $x_{n-1,k}$  will be outside the body, and fictitious values must be assigned for the dependent variables at this point in order to use the Lax numerical schemes. The present investigation shows that, for this situation, satisfactory results can be obtained by using a different numerical scheme. In the new scheme, the Lax method is applied in reverse to those dependent variables whose values are not specified on the boundary. Accordingly, a central difference formula is used for the time derivative and a forward difference formula for the space derivative. As an example, the space and time derivatives for the dependent variable  $V$  are

$$\begin{aligned}\frac{\partial V}{\partial \tau} &= \frac{V_{n,k+1} - V_{n,k-1}}{2\Delta\tau} \\ \frac{\partial V}{\partial x} &= \frac{V_{n+1,k} - \frac{1}{2}(V_{n,k+1} + V_{n,k-1})}{\Delta x}.\end{aligned}\tag{19}$$

However, for the derivatives of the dependent variables whose values are specified on the boundary,  $R$  in this case, the Lax scheme is kept. As a result, a fictitious value  $R_{n-1,k}$  must be assumed, which, together with the other two unknowns  $V_{n,k+1}$  and  $\Theta_{n,k+1}$ , can be obtained from the three finite-difference equations derived from Equations (10) or Equation (11)

using Equation (19). One of the unknowns in the calculation at an interior point,  $R_{n,k+1}$  is now a known quantity which has the specified value of  $R$  on the boundary at  $\tau = (k + 1)\Delta\tau$ .

#### 4. RESULTS FOR ELASTIC CASE - ACCURACY OF THE NUMERICAL SCHEME

The case of a constant pressure applied suddenly on the surface of a spherical cavity within an infinite elastic solid is studied to demonstrate the capability of the numerical scheme. In Figures 1a and 1b, the principal shear stress in the present numerical results is compared to those obtained analytically by Hunter<sup>8</sup> and numerically by Chou.<sup>9</sup> Chou uses the method of characteristics in obtaining his solution. Considering the rather large grid size (twenty-five points per cavity radius) used to obtain the present solution, the agreement between this solution and the other solutions is satisfactory.

However, near the elastic wave front where a jump in the values of the shear stress exists, a large discrepancy is observed between this solution and the exact one. The discrepancy is mainly due to the artificial viscosity which is associated with the numerical scheme and tends to smooth the sharp discontinuities of the solution. Since the artificial viscosity is equal to  $(1/2) (\Delta x)^2 / \Delta\tau$ , the effects can be greatly reduced by increasing the ratio of  $\Delta\tau / \Delta x$ , as indicated by comparing the results obtained with  $\Delta\tau / \Delta x = 1$  and  $1/2$ . For this case,  $\Delta x / \Delta\tau = 1$  is the maximum value allowed by the stability criterion; at this  $\Delta x / \Delta\tau$  ratio some small oscillations in the solution which are not shown in the figures have already occurred near the wave front. The effects of the viscosity can also be reduced by diminishing the grid size as seen in the results for  $\Delta x = 0.01$  and  $0.02$ . However, considering the increase of computing time, the second method of reducing viscosity is not as efficient as the first one. Roberts<sup>10</sup> has proposed a much more effective method for improving the accuracy of this numerical scheme. The technique involves combining several solutions of first order accuracy obtained with a fixed  $\Delta x / \Delta\tau$  ratio. Solutions with accuracy of order greater than one can be obtained very easily.

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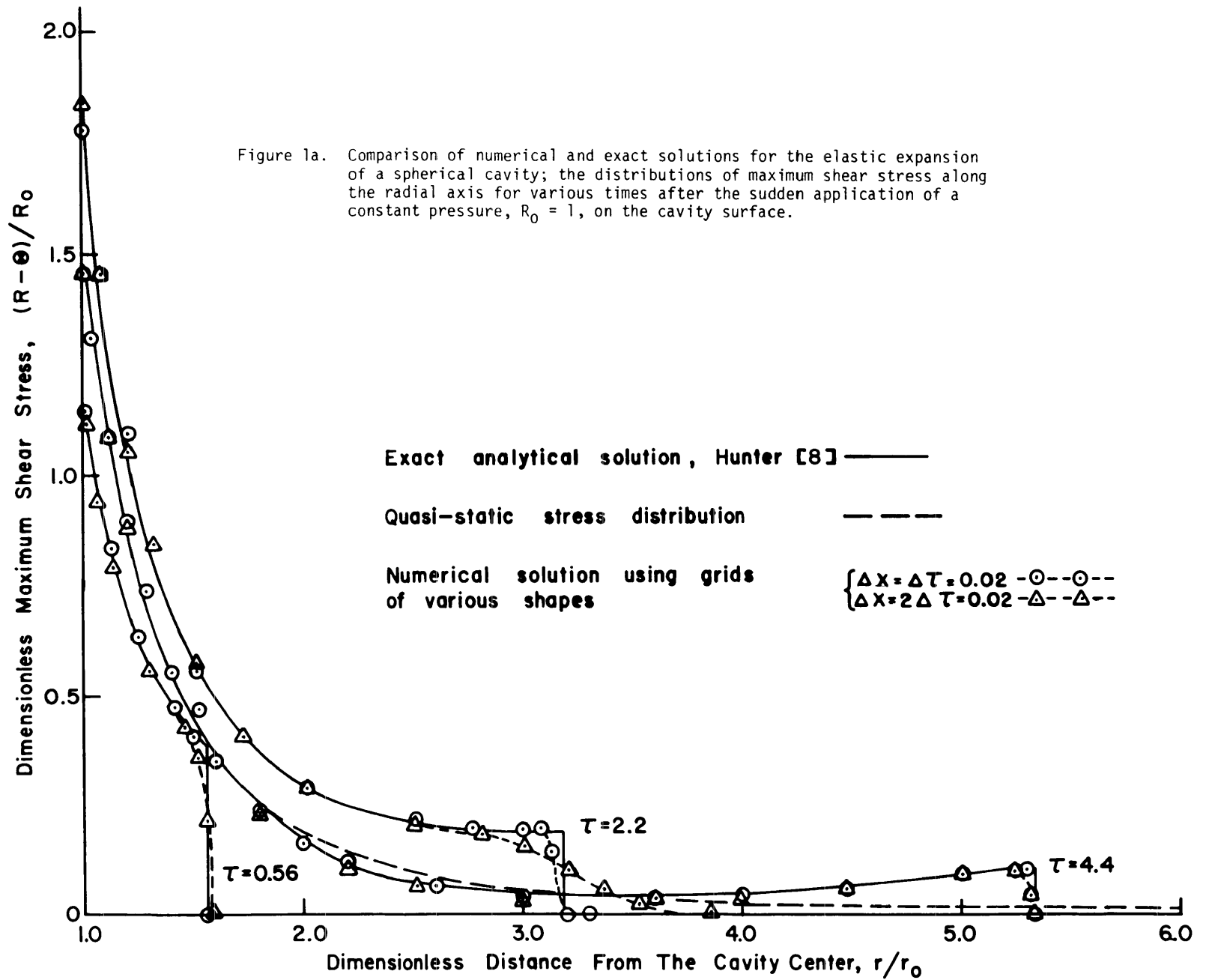
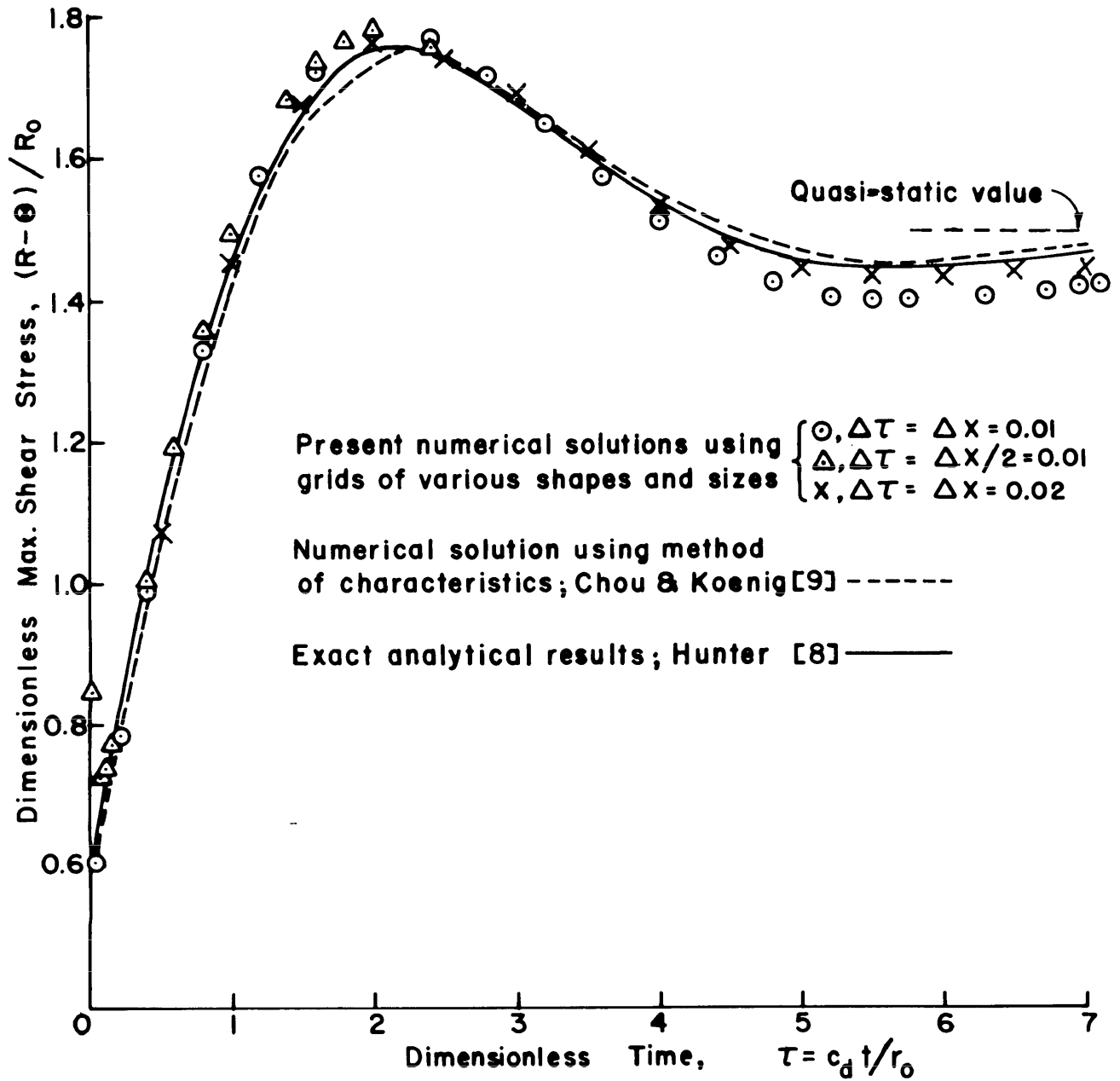


Figure 1b. Comparison of numerical and exact solutions for the elastic expansion of a spherical cavity; the variation of maximum shear stress at the cavity surface after the sudden application a constant pressure,  $R_0 = 1$ , on the surface.



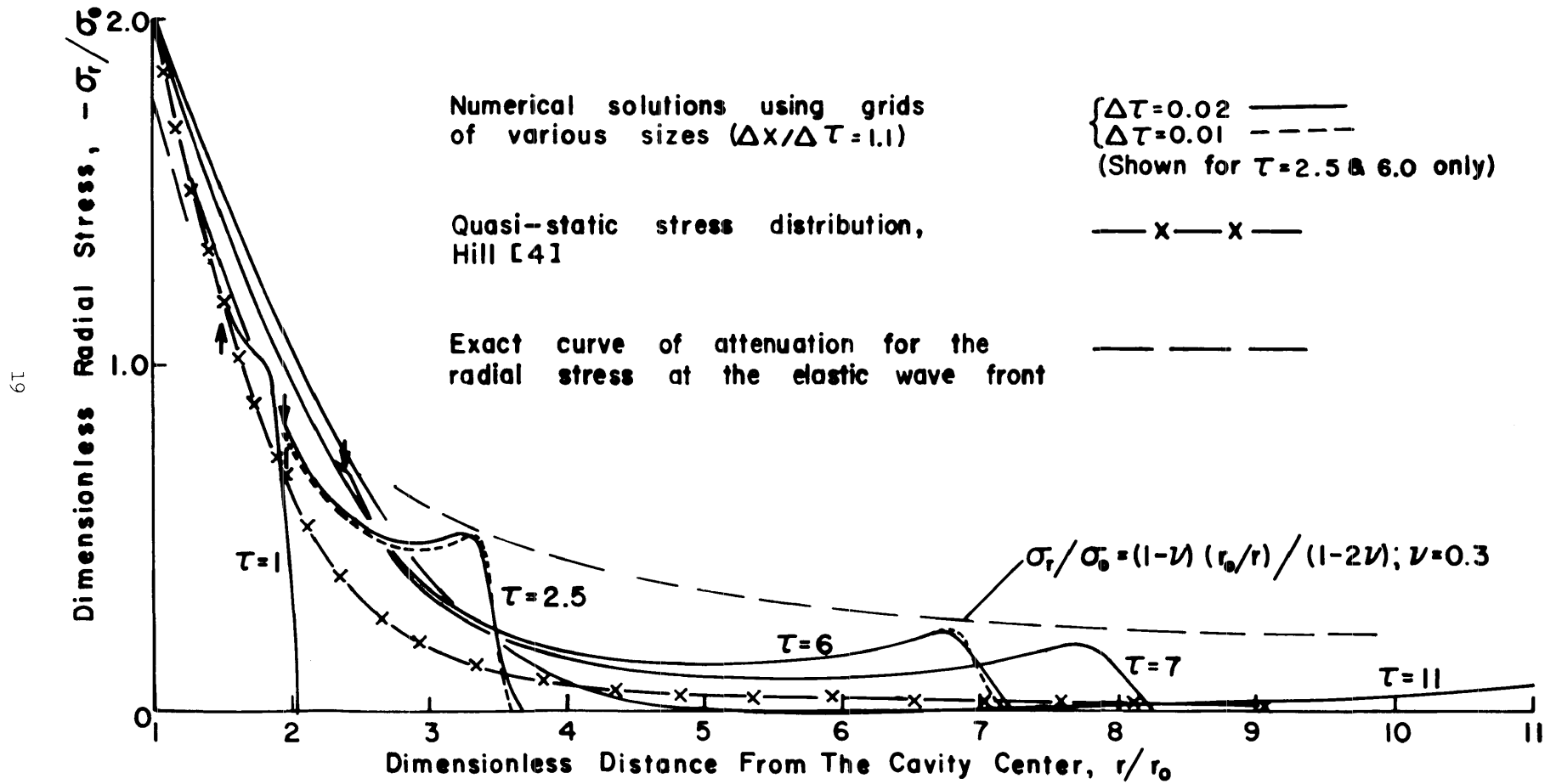


## 5. RESULTS FOR ELASTIC-PERFECTLY-PLASTIC MATERIAL

The numerical scheme just discussed has also been used to obtain the elastic-perfectly-plastic solution for a spherical cavity expanded under a constant internal pressure which is suddenly applied at  $\tau = 0$ . Typical sets of results are shown in Figures 2a through 3b for the cases in which the magnitudes of the internal pressures are 2 and 20 times of the constant yield stress of the material. In the results for the latter case, the evidence of two discontinuities can be seen in the profiles of stresses along the radial axis. The one which is farther away from the cavity surface is the elastic wave front while the one propagating behind is the plastic wave front which separates the regions of plastic and elastic deformation. The constant speed of propagation of the elastic wave front agrees closely with the exact value of the dilatation wave speed,  $c_d$  obtained here. The magnitude of the jump in the dependent variables across the elastic wave front also seems to agree with the exact solution which is represented by the dashed curves in Figures 2a to 2c.

The elastic-plastic boundary moves at a constant speed when a discontinuity in the radial stress across the boundary is visible near the beginning of the impact. The value of the speed agrees with the theoretical one predicted by Green,<sup>1</sup> namely,  $c_p = [E/3\rho(1 - 2\nu)]^{1/2}$ . This result is shown in Figure 4 in which the position of the elastic-plastic boundary at all times is plotted. Figure 4 also indicates that, for the present boundary condition, the movement of the elastic-plastic boundary will slow down as the discontinuity across the boundary disappears because of the geometrical dispersion. As long as the pressure on the cavity surface remains unchanged, the motion of the elastic-plastic boundary will come to a halt and then reverse its direction towards the cavity surface. This result is quite different from that observed in the static solution for the same elastic-plastic problem; when the constant pressure is applied quasi-statically at the boundary, a plastic zone next to the cavity surface is always maintained by the

Figure 2a. Numerical results for the elastic-plastic expansion of a spherical cavity; the distributions of radial stress along the radial axis for various times after the sudden application of a constant pressure,  $R_0 = 2$ , on the cavity surface. Arrows indicate the positions of the elastic-plastic boundary.



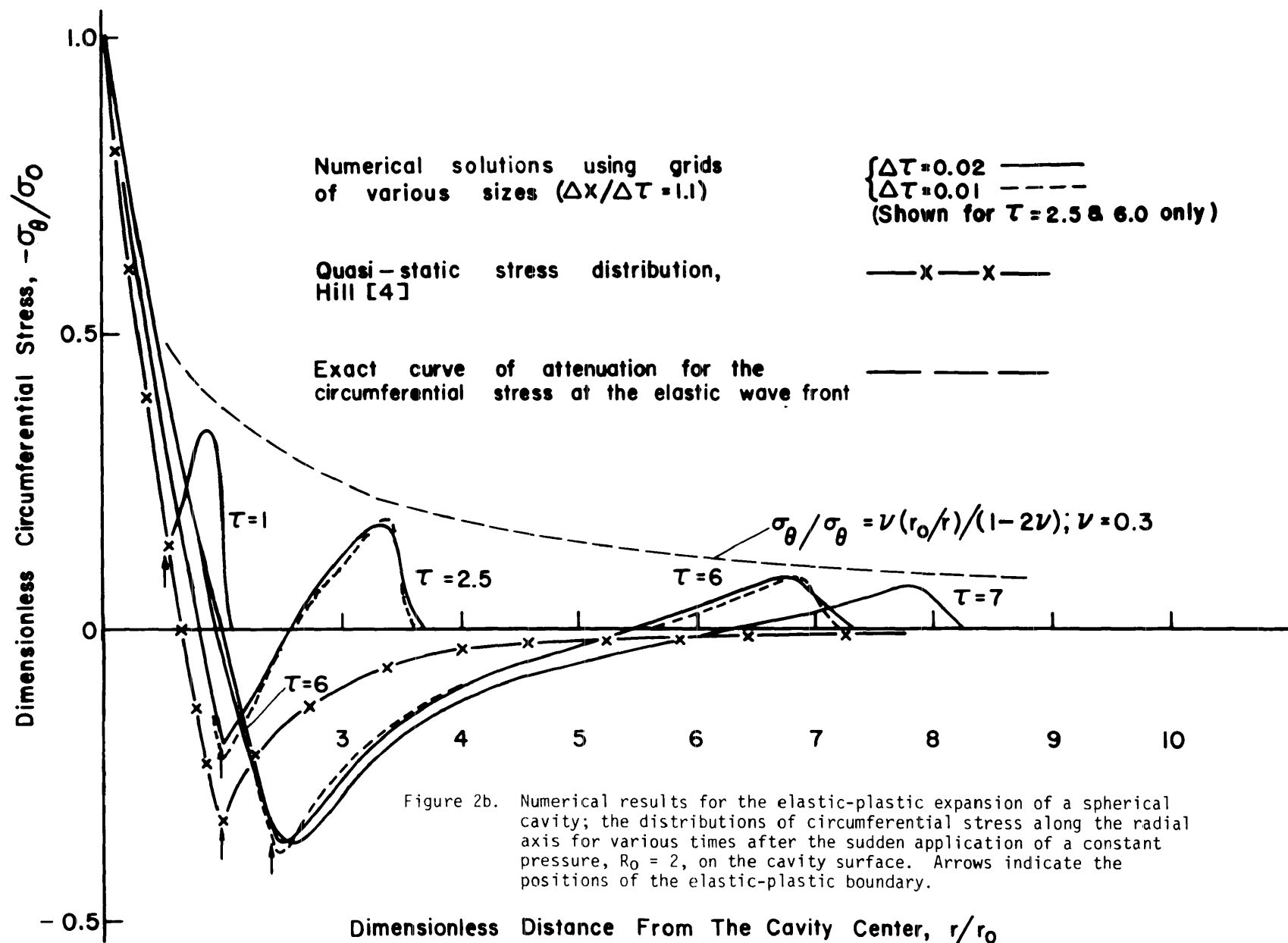


Figure 2c. Numerical results for the elastic-plastic expansion of a spherical cavity; the distributions of particle velocity along the radial axis for various times after the sudden application of a constant pressure,  $R_0 = 2$ , on the cavity surface. Arrows indicate the positions of the elastic-plastic boundary.

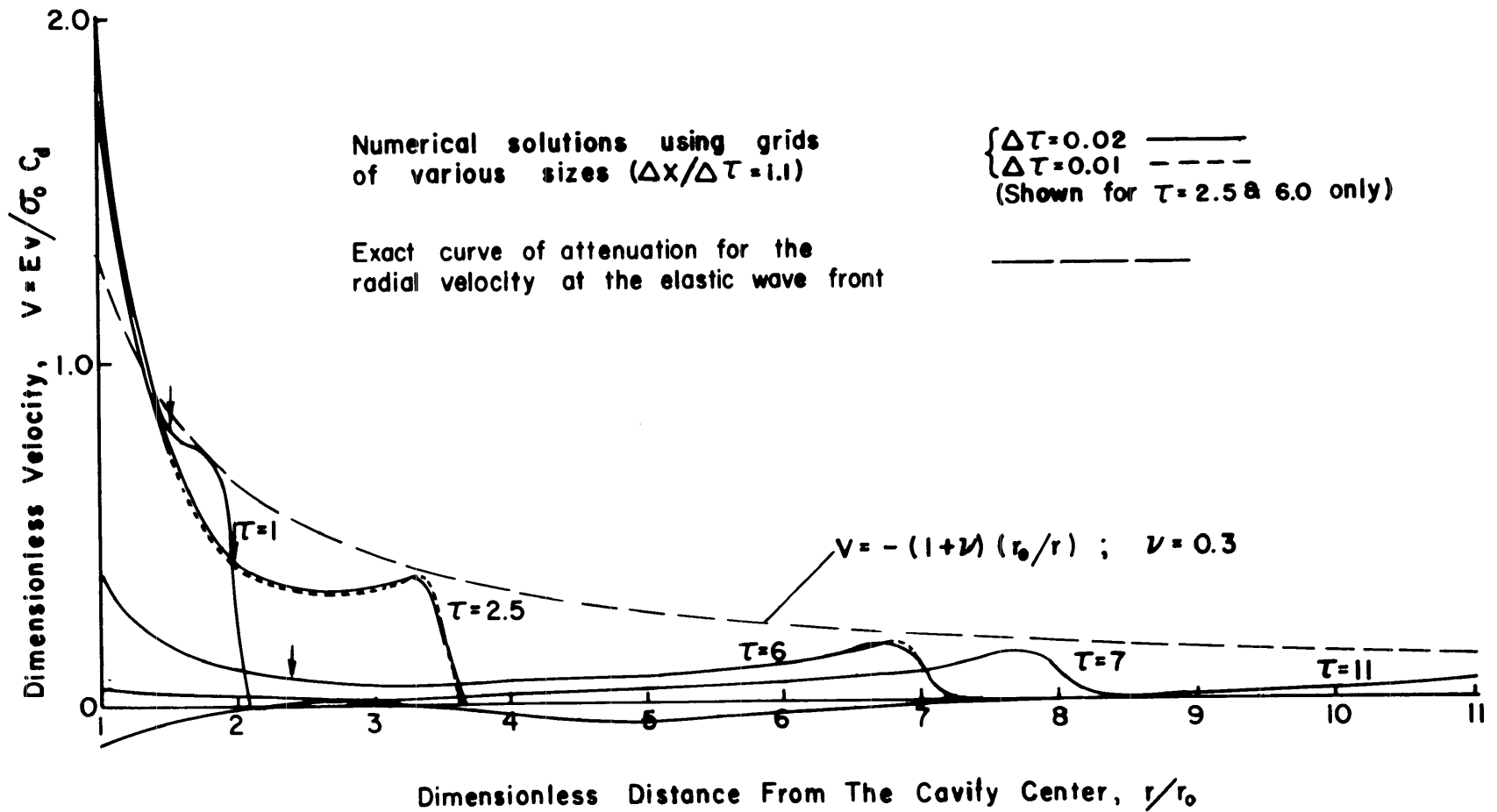


Figure 2d. Numerical results for the elastic-plastic expansion of a spherical cavity; the distributions of radial displacement along the radial axis for various times after the sudden application of a constant pressure,  $R_0 = 2$ , on the cavity surface. Arrows indicate the positions of the elastic-plastic boundary.

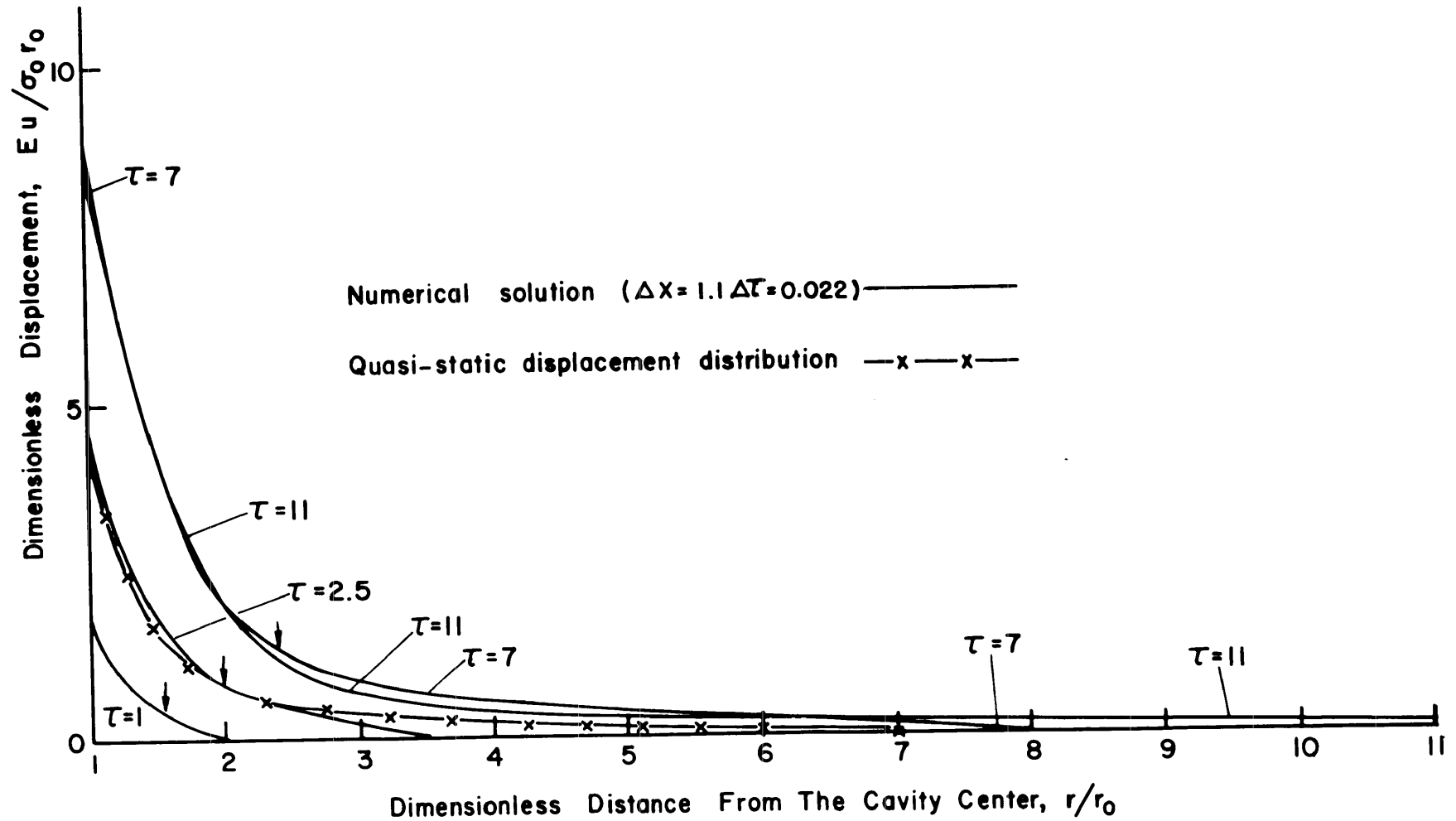


Figure 3a. Numerical results for the elastic-plastic expansion of a spherical cavity; the distributions of radial stress along the radial axis for several times after the sudden application of a constant pressure,  $R_0 = 20$ , on the cavity surface. Arrows indicate the positions of the elastic-plastic boundary.

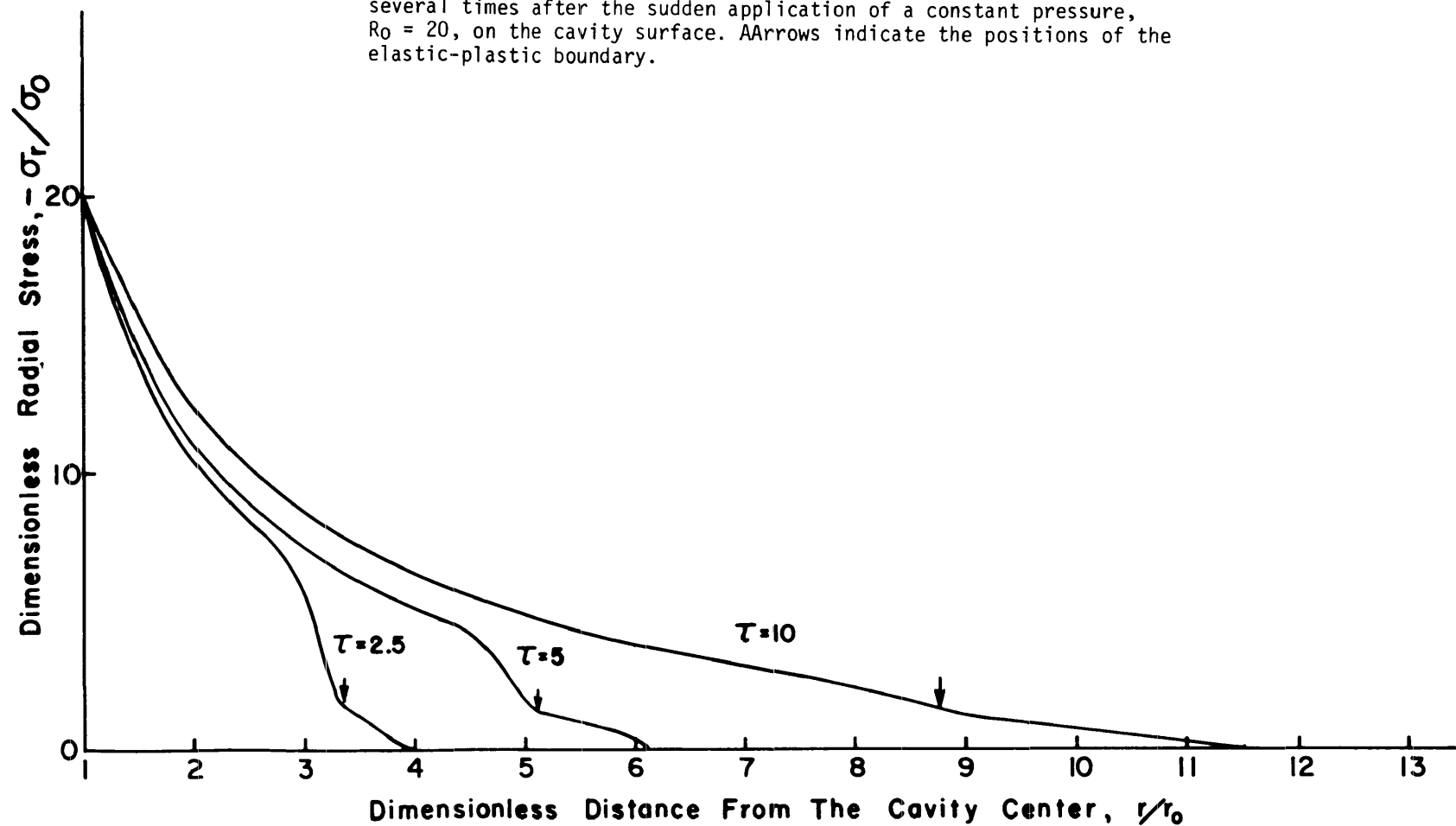
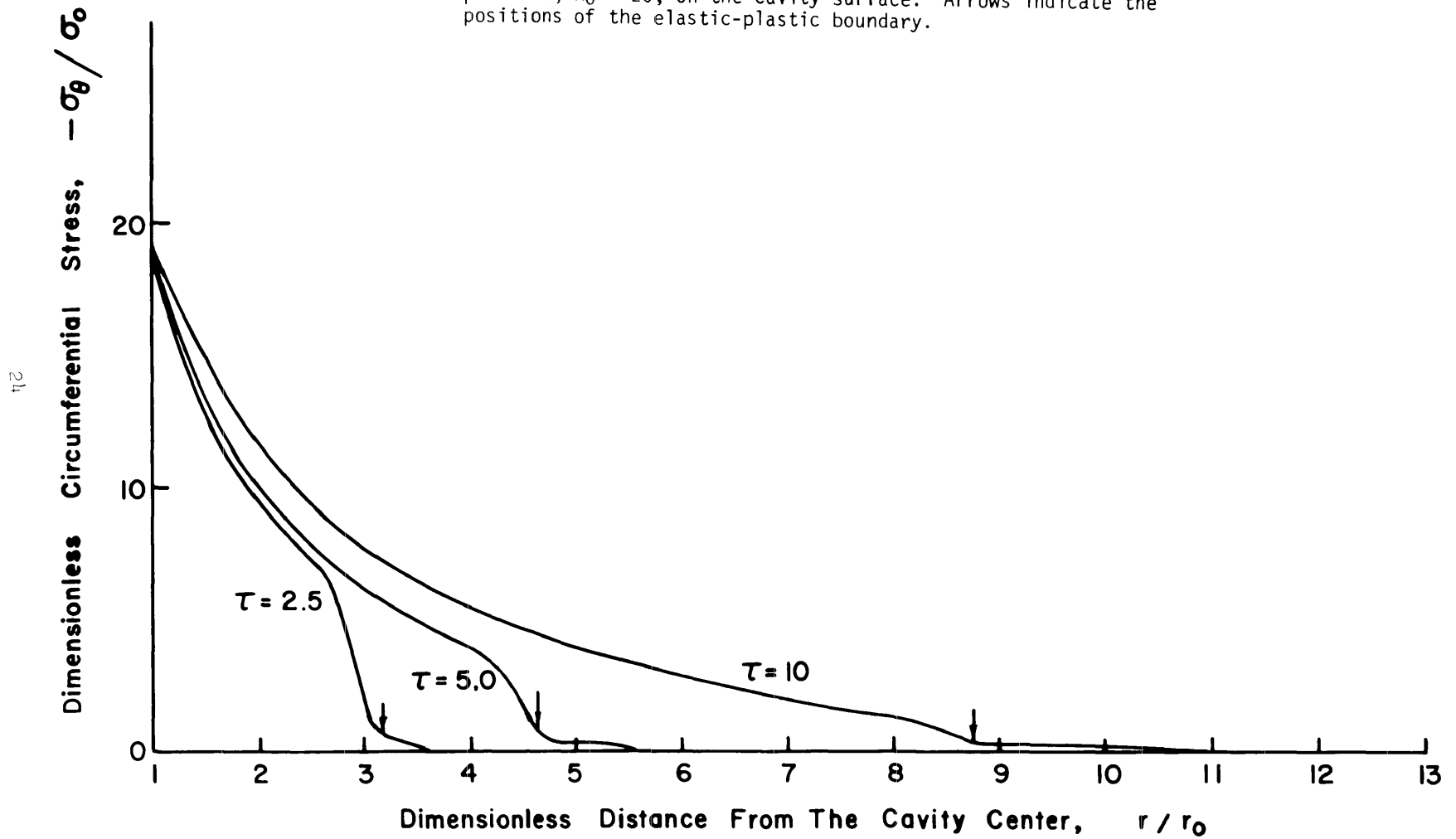
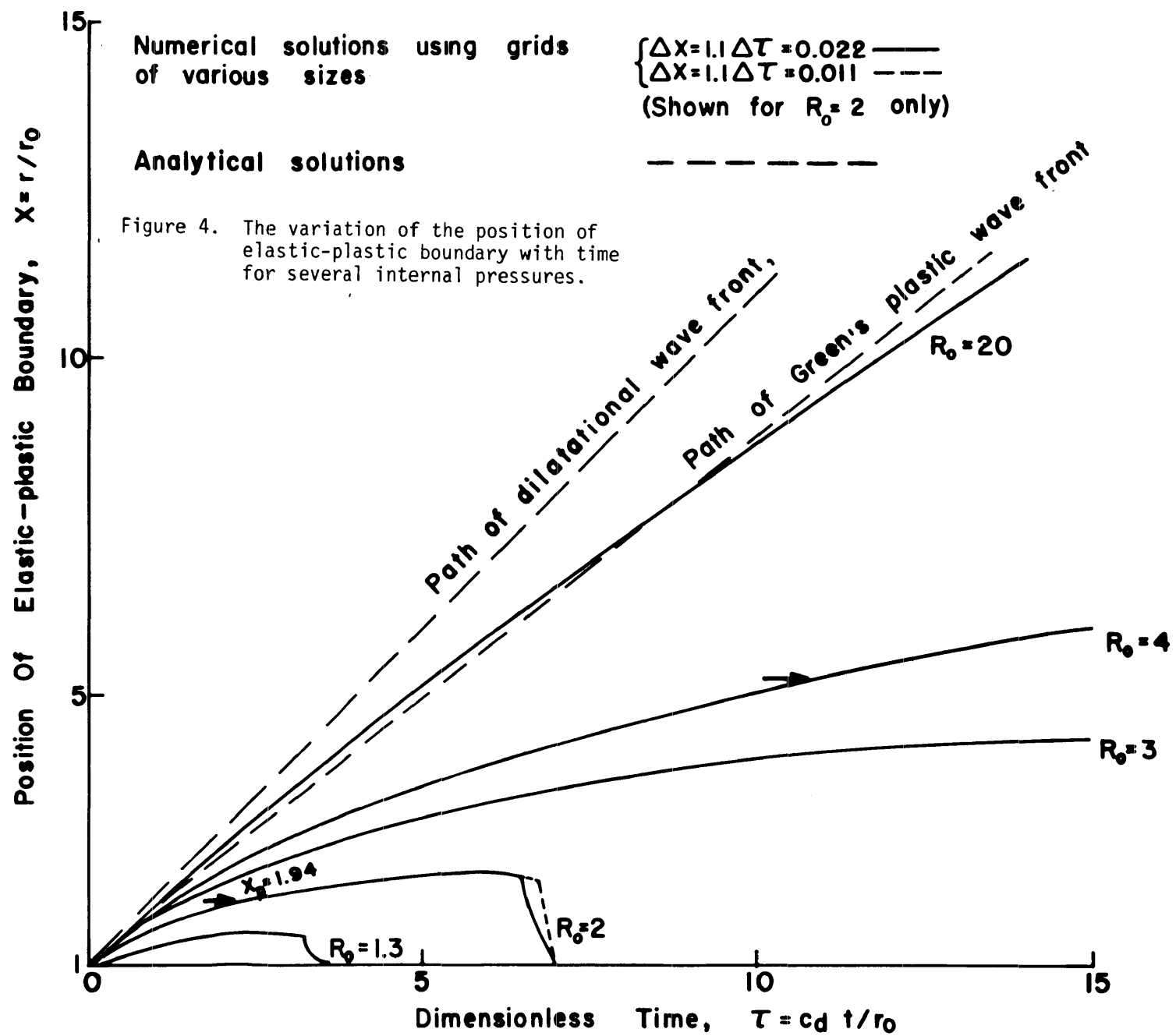


Figure 3b. Numerical results for the elastic-plastic expansion of a spherical cavity; the distributions of circumferential stress along the radial axis for several times after the sudden application of a constant pressure,  $R_0 = 20$ , on the cavity surface. Arrows indicate the positions of the elastic-plastic boundary.

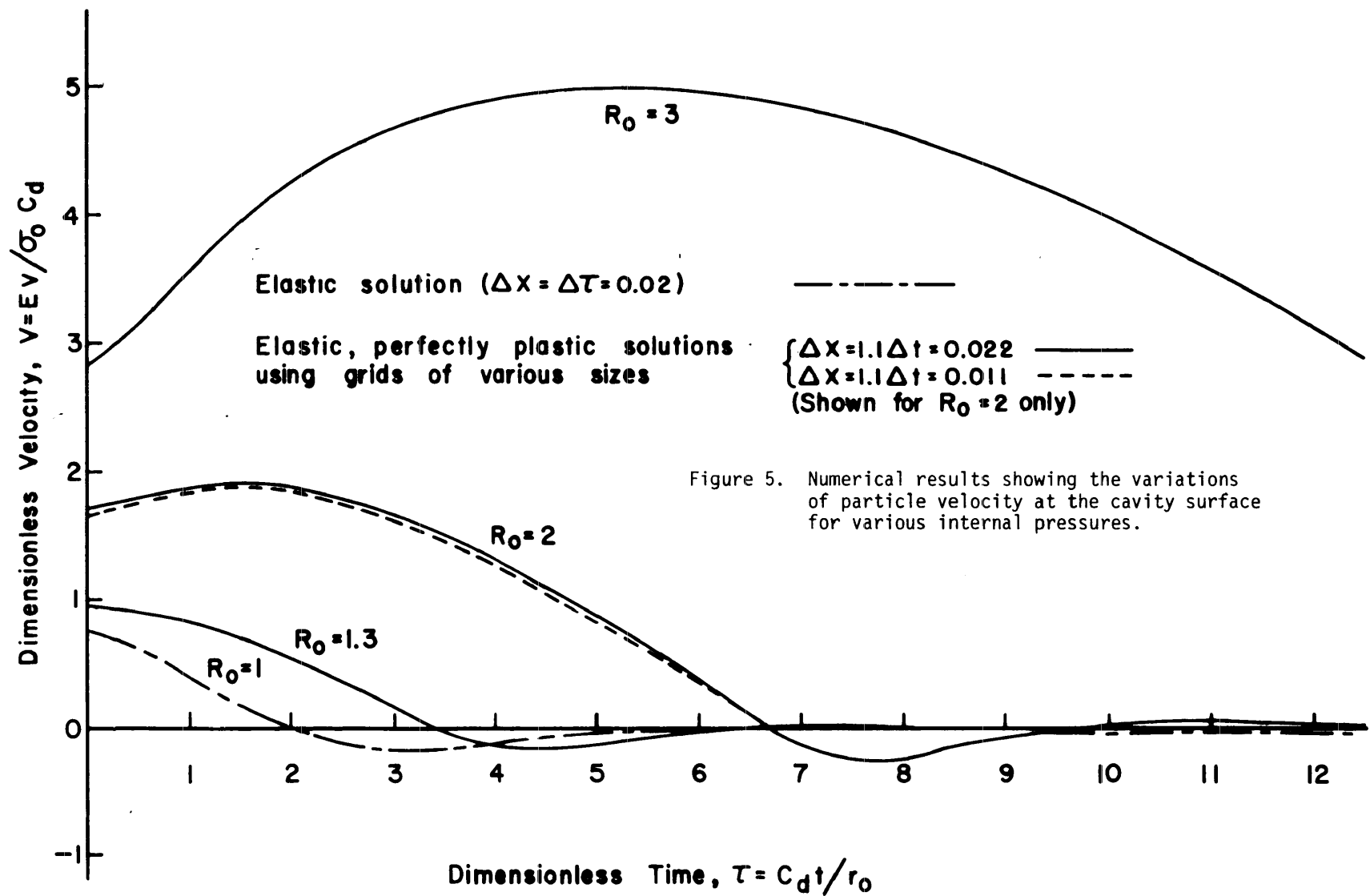






applied pressure.<sup>5</sup> Differences between the static solutions and the asymptotic dynamic solution at large times can also be observed in the distributions of the stresses and displacements along the radial axis (Figures 2a to 2d). A larger deformation and a higher radial stress occur in the dynamic case than do in the static case. These differences are not likely to be attributable to errors in the numerical solution, because the comparison of the results obtained with two grid sizes does not suggest the possible existence of such large differences in the solution (Figure 2a). The maximum radius of the elastic-plastic boundary is also larger in the dynamic case than in the static case. These disagreements between the dynamic solution at a large time and the static solution are probably due to the residual stresses produced by the inertial force occurring in the dynamic deformation process. This result implies that, in some circumstances, the quasi-static approximation may not be valid in dynamic plasticity. The quasi-static approximation in which the static solution of a problem is used to describe the long-time behavior of the dynamic solution holds in many situations in dynamic elasticity.<sup>8,11,12</sup>

Figures 4 and 5 show, for all times, the dependence of the particle velocity and of the position of the elastic-plastic boundary on the magnitude of the applied pressure. Although all of the results for various applied pressure seem to be similar qualitatively, the magnitudes of various parameters in the solution seems to be quite sensitive to a change in the pressure. The dependence is rather nonlinear. It is also interesting to note that the nonlinearity also appears in the result of the total duration of the dynamic deformation, namely, the duration during which the acceleration of the material is not vanishingly small. This duration for the elastic-plastic problem increases with increasing applied pressure, while, in the elastic case, the duration is a constant as shown in Figure 5. Accordingly, the initiation of slight plastic deformations tend to change considerably the nature of the flow.



## 6. CONCLUSIONS

It has been shown that the governing equations for an elastic-perfectly-plastic flow with spherical symmetry can be solved through the use of a finite difference technique whose nature is rather well understood in comparison to other similar methods. With a slight modification of the original scheme proposed by Lax, the method can be applied to solve a wide variety of mixed initial-and boundary-value problems involving elastic and plastic flows. The comparison of the results for the dynamic deformation of an elastic cavity obtained using this method and an exact analytical method shows that the accuracy of the numerical solution is acceptable, except probably in the region where a discontinuity in the dependent variables appears. Large errors may occur in this region because of the artificial viscosity added into the equations by using the numerical scheme. Accordingly, for solutions containing discontinuities, an investigation of the error is necessary by varying the grid size and  $\Delta x/\Delta t$  ratio. If high accuracy is required for the solution near the discontinuities, it is advisable to locate and obtain the discontinuities in the solution using an analytical method as shown by Friedman et al.<sup>4</sup>

For the dynamic expansion of a spherical cavity in an elastic-perfectly-plastic solid, the numerical results for a suddenly applied internal pressure confirm Green's analytical prediction that an elastic-plastic boundary propagates with a constant speed,  $c_p = [E/3\rho(1 - 2\nu)]^{1/2}$ , when discontinuities in stress and particle velocity occur across the boundary. Within the major duration of an impact the elastic-plastic boundary moves with a variable speed, of which the magnitude cannot be predicted analytically. Therefore, a numerical technique seems to be unavoidable in the investigation of spherical elastic-plastic flow.

The same results have also shown that the quasi-static approximation which is often used in dynamic elasticity may not necessarily hold in dynamic plasticity. The solution for the dynamic problem investigated

does not approach the static solution as a limit when time elapses. This result is probably due to the residual stress produced by the inertia force of the impact.

As far as magnitudes are concerned, the results show a rather nonlinear dependence of the elastic-plastic solution of the internal pressure. This nonlinearity seems to indicate that the nature of the flow would be changed considerably by the initiation of a slight amount of plastic deformation.

#### ACKNOWLEDGEMENTS

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## APPENDIX

### A COMPUTER PROGRAM FOR THE DYNAMIC EXPANSION OF A SPHERICAL CAVITY BY AN INTERNAL PRESSURE (ELASTIC-PERFECTLY-PLASTIC SOLUTION)

The computer program used for obtaining the present results is attached at the end of this Appendix. The program was written in the FORAST language by Mr. T. Addison of the Computing Laboratory. (The reader may consult one of the references at the end of the Appendix for a description of the computer language.) The program can be executed with the BRLESC Computer which is currently being used in the Ballistic Research Laboratories.

The computer program is based on the finite difference schemes described in the main text of this report and will be able to provide information of stress, displacement, and velocity around a spherical cavity which is being expanded by an internal pressure specified with card numbers 157 and 171. The pressure at the boundary is shown as a constant,  $RR$ , but can be changed to a function of time,  $f(\tau)$ ;  $\tau$  is represented by  $T$  in the program. The physical quantities  $R$ ,  $\theta$ ,  $V$ , and  $U$  are represented by  $R$ ,  $Q$ ,  $V$  and  $U$ , respectively.

The program can be used for problems with non-vanishing initial values, if the section between cards 13 and 39 is properly modified. This section is now written to represent the similarity solution near  $\tau = 0$  for this problem. The similarity solution was put in the program as an alternate way of calculating the results but was omitted in obtaining the final results presented in this report.

The numerical calculation proceeds in increments of time,  $\Delta\tau$ , while at a given time,  $\tau$ , it begins with the two data points nearest to the boundary,  $x = 1$  and shifts in the positive  $x$  direction. Calculations at points other than the boundary point are shown in cards 60 to 79; in cards 60 to 65, the plastic equations are used, and in cards 72 to 79, the elastic equations are applied. Computations at the boundary point occur only once every other time step, as shown in cards 155 to 170 for an elastic case and in cards 171 to 182 for a plastic case.

The input data concerning material properties, specifications for outputs and the dimensions of the grid used for calculations are punched in the two cards at the end of the program which will be read by the machine upon the order written in cards 9 and 11. The input data include the following information:

Program Notations	vs	Physical Quantities
DX:		$\Delta x$
DT:		$\Delta t$
DTT:		t of the first output
DTI:		interval of t for output
TMAX:		t of terminating calculation
RR:		Radial stress at the boundary, $x = 1$
NU		$v$
RTO:		initial value of radial stress
A:		$\bar{\sigma}$
TO:		initial time.

The output of the calculation will be stored in a magnetic tape and can be tabulated in a numerical form (cards 112 to 126) as well as plotted in a graphical form (cards 134 to 147). The format of printing is indicated in cards 16 and 17. In plotting, the distributions of stresses, particle velocity and displacement along the radial axis will be presented by color curves in a single graph for each time, t. The size and scaling of the graph is controlled with cards No. 128 and 129. The curves of R,  $\theta$ , V and U are all referred to the same scale which is chosen for R, and hence, it is sometimes necessary to adjust the values of V and U with proper numerical factors before plotting, as shown in cards 132 and 149.

		1
		2
%	MAXT(35) MIN	3
%	MAXO(95000) LINES	4
	BLUC(Y-Y5000)AH1-AH444)U0-U5000)V0-V5000)R0-R5000)Q0-Q5000)	5
	BLUC(I1-I12)D1-D2)	6
	SYN (Y=X1)	7
START	HEAD=(FW2)-(D1)D2)	8
	READ(DX)DT)DIT)DII)TMAX)RM)	9
	ENTER(SET(IU)8)0)U)100)	10
	READ(NU)X1)RT0)A')T0)	11
	HEAD=(F1)-(12)NOS.AT(I1)%	12
IST	INI(K=1)% INI(KK=12+1)% X=X1% INT(N=4)% I=TU%	13
	C2D2=(1-NU-2*NU**2)/(1-NU)% R=RT0% CUCD=SQRT(C2D2)	14
	CN=(1-NU)/3(1+NU)(1-2*NU)**2% CN=SQRT(CN)	15
	IF(R(1-NU)/(1-NU))> 1)GOTO(3.0)	16
1.5	R1,I=R10% Q1,I=(NU/(1-NU))R% V1,I=- A'*R*C2D2	17
	MM=1%	18
	COUNT(5)IN(1)GOTO(1.5)% INT(P=1)	19
	GOIO(PUN)	20
3.0	IF(X>CV*T0)GOTO(4.0)	21
	R1,I=R10% Q1,I=R1,I)-1%	22
	V1,I=-A'((1+NU)-(3(1-NU)**3/(1+NU)(1-2*NU)**2)**.5	23
	CONT +(3(1-NU)/(1+NU))**.5*R1,I	24
	X=X+2*DX% MM=2.0% COUNT(5)IN(1)GOIO(3.0)	25
	GOIO(PUN)	26
4.0	R1,I=(1-NU)/(1-2*NU)% Q1,I=NU/(1-2*NU)% V1,I=-(1+NU)A'	27
	X=X+2*DX% MM=3.0%	28
	COUNT(5)IN(1)GOTO(3.0)	29
PUN	CLEAR(5)NOS.AT(R1)% CLEAR(5)NOS.AT(V1)% CLEAR(5)NOS.AT(Q1)	30
PUN	PUNCH<INITIAL CONDITIONS>% ENTER(PUNCH B)	31
	PUNCH< I X V M >	32
CONT	U C I>	33
	SET(I=0)J=1)% CC1=D1% CC2=D2	34
PUN1	PUNCH=(FW5)-(CC1)CC2)J)X)V1,I)R1,I)Q1,I)J)	35
	CC1=CC2=0	36
	X=X+2*DX% INC(J=J+1)% COUNT(5)IN(1)GOTO(PUN1)	37
	SET(I=0)	38
	PUNCH<EJECT PAGE>	39
COMM	COMPUTE CONSTANTS	40
	CCNU=(1-NU-2*NU**2)	41
	A=1/(1-NU-2*NU*NU)A'% 2DT=1/2*DT% 1N=1-NU	42
	B11=2DT*1N*A% B12=1/2*DX-1% B21= NU/DX-1% B23=1 /A*DT	43
	B31=A(.5*1N/DX-NU)% B32=-2DT	44
	A11=2DT% A22=-2DT% 2DX=1/2*DX% A12=A'*C2D2*2DX	45
	A21=(2DX-1)(1/3*A'(1-2*NU))	46
	SET(BY=1)	47
4.1	N1=1% N2=2% I=1+DT %SET(I=0)	48
	INC(K=K+1)	49
	IF-INI(K=2(K/2)) GOTO(EVEN)%	50
	VK2=V1% QK2=Q1% RK2=R1 % SET(AY=1)GOIO(4.2)	51
EVEN	INI(N=N+1)% SET(AY=2)	52
	VK1=V1% QK1=Q1% RK1=R1% UK1=U1	53
4.2	GOIO,AY(AY1)AY2)	54
AY1	XN=X1+N2*DX% N2=N2+2% GOTO(4.5)	55
AY2	XN=X1+N1*DX% N1=N1+2	56
4.5	C1=D1((1-2*NU)(V2,I-V1,I)/2*DX+(NU-.5)(V2,I+V1,I)/XN)/	57
	CONT CCNU*A'+.5(K2,I+R1,I)-.5(Q2,I+U1,I)	58
	IF-ABS(C1<1.)GOIO(EQ2)	59



	COMM EQUATION 1 PLASTIC	80
	IV =DI*A'*C2D2((R2,I-R1,I)/2*DX+2/XN)+(V2,I+V1,I).5	81
	R1,I=DI((V2,I-V1,I)/2*DX+(V2,I+V1,I)/XN)/3(1-2*NU)*A'+	82
	CONT .5(R2,I+R1,I)	83
	U1,I=R1,I-1	84
	U1,I=(DT(V2,I+V1,I)+U2,I+U1,I).5	85
	V1,I=IV	86
	IP2(U50)(04L)(V1,I)%% 0 GOES TO BIT 9 OF V1	87
	COMM ZERO GOES TO BIT 9 OF V1=PLASTIC EQ.	88
	COUNT(N)IN(I)GOTO(4.2)	89
	IF-INT(K=2(K/2))GOTO(8.8)% GC10(BOND)	90
	COMM EQUATION 2 ELASTIC	91
EQ2	IV=C2D2*A'*DI((R2,I-R1,I)/2*DX+(R2,I+R1,I-Q2,I-U1,I)	92
	CONT /XN)+(V2,I+V1,I).5	93
	R1,I=DI(1N(V2,I-V1,I)/2*DX+NU(V2,I+V1,I)/XN)/(CCNU)*A'	94
	CONT +.5(R2,I+R1,I)	95
	U1,I=DI(NU(V2,I-V1,I)/2*DX+(V2,I+V1,I)/2*XN)/CCNU*A'	96
	CONT +.5(Q2,I+U1,I)	97
	U1,I=(DT(V2,I+V1,I)+U2,I+U1,I).5	98
	V1,I=IV	99
	IP2(U4L)(04L)(V1,I)%	100
	COMM ONE GOES TO BIT 9 OF V1=ELASTIC EQ.	101
	COUNT(N)IN(I)GOTO(4.2)%	102
	IF-INT(K=2(K/2))GOTO(8.8)% GC10(BOND)	103
8.8	SEI(I=U)J=1)% X=X1% CC1=D1% CC2=D2	104
	IF-INT(K> 11) GOTO(9.5)	105
	IF-INT(K=2(K/2))GOTO(4.1)	106
	PUNCH-(FW1)-< TIME= >T< CYCLE NO. >(K )	107
	ENTER(PUNCH B)	108
	PUNCH<	109
	CONT< U U I>	110
9.0	SHX(V1,I)(R8)(MM)	111
	COMM BIT NINE GOES TO BIT ONE	112
	B8(MM)(04L)(MM)	113
	COMM BIT ONE ONLY RESULTS IN MM=0 OR 1	114
	IP2(U50)(04L)(V1,I)%% REPLACES BIT 9	115
	PUNCH-(FW5)-(CC1)CC2)J)X)V1,I)R1,I)Q1,I)U1,I)MM)	116
	X=X+2*DX% CC1=CC2=0 % INC(J=J+1)	117
	COUNT(N+1)IN(I)GOTO(9.0)% PUNCHKEJEC PAGE>	118
	GOTO(4.1)	119
	IF-INT-NOT(K=KK)GOTO(10.1)	120
9.5	IF-NOT( T=DT1)WITHIN(DT )GOTO(10.1)	121
	DT1=DT1+DT1	122
	IF-INT-NOT(K=2(K/2))GOTO(PUN1 )	123
	CC1=CC2=0% X=X+DX	124
	PUNCH-(FW1)-< TIME= >T< CYCLE NO. >(K )	125
	ENTER(PUNCH B)	126
	PUNCH<	127
	CONT< U U I>	128
8.9	SHX(V1,I)(R8)(MM)	129
	B8(MM)(04L)(MM)	130
	IP2(U50)(04L)(V1,I)	131
	PUNCH-(FW5)-(CC1)CC2)J)X)V1,I)R1,I)Q1,I)U1,I)MM)	132
	X=X+2*DX% INC(J=J+1)	133
	COUNT(N )IN(I)GOTO(8.9)% GOTO(PEP)	134
	INT(KK=KK+12)	135
PUN1	PUNCH-(FW1)-< TIME= >T< CYCLE NO. >(K )% ENTER(PUNCH B)	136
	PUNCH<	137
	CONT< U U I>	138
9.7	SHX(V1,I)(R8)(MM)	139

	COMM BIT NINE GOES TO BIT ONE	120
	BB(MM)(04L)(MM)	121
	COMM BIT ONE ONLY RESULTS IN MM=0 OR 1	122
	IP2(U50)(04L)(V1,I)%% REPLACES BIT 9	123
	PUNCH-(FW5)-(CC1)CC2(J)X)V1,I)R1,I)U1,I)MM)	124
	X=X+2*DX% CC1=CC2=0 % INC(J=J+1)	125
	COUNT(N+1)IN(I)GOTO(9.7)%	126
PEP	PUNCH<EJECT PAGE>	127
	XMAX=14.0% RMAX=15.% XMIN=1.0% RMIN=-4.5	128
	RX=RS=RMAX/10% XX=XS=(XMAX-XMIN)/26	129
6.60	SEL(I=0)%	130
	Y=X1	131
6.61	V1,I=-V1,I/5.0% Y1,I=Y,I+2*DX% U1,I=-U1,I/500.	132
	COUNT(N+1)IN(I)GOTO(6.61)	133
	GOTO,BY(AY5)(AY6)(AY3)(AY4)	134
AY5	SEL(XB=4)(YB=14)%	135
AY7	INC(BY=BY+1)GOTO(9.8)	136
AY6	SEL(XB=31)GOTO(AY7)	137
AY3	SEL(YB=42)GOTO(AY7)	138
AY4	SEL(XB=4)(BY=1)%	139
9.8	STOR	140
	PUNCH<XB= >XB < YB= >YB	141
	ENTER(PLOT.S)1),XB),YB)051)050)XS)RS)AH1))	142
	ENTER(PLOT.D)1)1)1)3)Y)Q1),I)	143
	ENTER(PLOT.D)1)1)1)4)Y)R1),I)	144
	ENTER(PLOT.D)1)1)1)5)Y)U1),I)	145
	ENTER(PLOT.D)1)1)1)6)Y)V1),I)	146
	ENTER(PLOT.A)1)1)XX)RX)XMIN)XMAX)RMIN)RMAX)	147
	SEL(I=0)	148
6.62	V1,I=-V1,I/5.0% U1,I=-U1,I/500	149
	COUNT(N+1)IN(I)GOTO(6.62)	150
9.9	IF-INI(BY>1)GOTO(10.1)% ENTER(PLOT.F)1)	151
10.1	IF(T>IMAX)GOTO(NPROB)	152
	IF-INI(K=13)GOTO(NPROB)% GOTO(4.1)	153
NPROB	ENTER(ENDPLOT)1)% GOTO(N.PROC)	154
BOND	SEL(I=0)	155
	B14=2DT*1N*A*VK1+(.5/DX+1)RK2-QK1	156
	R0=RM	157
	B24=QK1/A*DI+2*NU*VK2/DX-(NU/LX-1)VK1	158
	B34=-RU/DT+2DT*RK2+A(1N*VK2/DX-(.5*1N/DX-NU)VK1)	159
	V0=(B32(B23*B14-B24)-B34*B23*B12)/(B32(B23*B11-B21)	160
CONT	-B31*B23*B12)	161
	U0=(B24-B21*V0)/B23	162
	IF-ABS(R0-U0>1)GOTO(BOND1)	163
	U0=(VU+VK1)DI+UK1	164
	MOVE(N)NOS.FROM(VU,N/-1)TC (V1,N/-1)	165
	MOVE(N)NOS.FROM(R0,N/-1)TC (R1,N/-1)	166
	MOVE(N)NOS.FROM(U0,N/-1)TC (U1,N/-1)	167
	MOVE(N)NOS.FROM(U0,N/-1)TC (U1,N/-1)	168
	V1=V0% Q1=U0% R1=R0% U1=U0	169
	GOTO(8.8)	170
BOND1	R0=RM	171
	A23=-R0-.5*RK2)/DT+(VK2/LX-(.5DX-1)VK1)/5*A*(1-2*NU)	172
	A13=2DT*VK1+A'*C2D2(2DX*RK2+2)	173
	V0=(A22*A13-A12*A23)/(A11*A22-A21*A12)	174
	U0=R0-1	175
	U0=(VU+VK1)DI+UK1	176
	MOVE(N)NOS.FROM(VU,N/-1)TC (V1,N/-1)	177
	MOVE(N)NOS.FROM(R0,N/-1)TC (R1,N/-1)	178
	MOVE(N)NOS.FROM(U0,N/-1)TC (U1,N/-1)	179

	MOVE(N)NOS.FROM(U0,N/-1)TC(U1,N/-1)	180
	V1=V0% Q1=Q0% R1=R0% U1=U0	181
	GO10(8.8)	182
F1	FORM(4-6)1-12)	183
FW1	FORM(12-10)4-6)	184
FF	FORM (12-6)3-1)1-10)	185
FW2	FORM(8-10)8-10)	186
FW5	FORM(8-10)8-4)4-6)3-2)1-1)12-10)3-1)1-5)4-6)	187
	LIST	188
	END GOTO(START)	189
BOUNDARY PI,		
.011	.01	.25
30000000 U0	10000000 U1	0.10
	50000	
PRUB		

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